See published version at https://doi.org/10.1016/j.visres.2020.10.003for

Stereoscopic depth constancy from a different direction

Robert S. Allison and Laurie M. Wilcox
Centre for Vision Research, York University

Abstract

To calibrate stereoscopic depth from disparity our visual system must compensate for an object's egocentric location. Ideally, the perceived three-dimensional shape and size of objects in visual space should be invariant with their location such that rigid objects have a consistent identity and shape. These percepts should be accurate enough to support both perceptual judgments and visually-guided interaction. This brief note reviews the relationship of stereoscopic depth constancy to the geometry of stereoscopic space and seemingly esoteric concepts like the horopter. We argue that to encompass the full scope of stereoscopic depth constancy, researchers need to consider not just distance but also direction, that is 3D egocentric location in space. Judgements of surface orientation need to take into account the shape of the horopter and the computation of metric depth (when tasks depend on it) must compensate for direction as well as distance to calibrate disparities. We show that the concept of the horopter underlies these considerations and that the relationship between depth constancy and the horopter should be more explicit in the literature.

Keywords: Stereopsis; depth constancy; horopter; depth scaling; depth perception; distance

1 Introduction

1.1 Visual Constancies

One of the most impressive feats of human perception, arguably its major achievement (Gillam, 2000), is that of perceptual constancy. That is, our ability to maintain a relatively constant perception of an object's properties despite variations in its retinal attributes due to factors such as orientation, illumination and position in space. Perhaps the most familiar and widely studied of these is size constancy – the tendency for an object to retain its apparent size despite changes in distance to the observer. A related, but in recent times somewhat under-appreciated, constancy is that of depth constancy. Both of these constancies involve maintaining the consistency of judgments of object dimensions over changes in distance. While size constancy refers to scaling the extent of the object in the fronto-parallel plane at different distances; depth constancy refers to scaling extent along the sagittal plane at different distances. To borrow an example from Ono and Comerford's (1977) review, size constancy occurs if a pencil viewed from the side appears to have the same length at different distances; depth constancy occurs when the pencil is rotated to point at you, and has the same apparent length at different distances. Importantly, these constancies differ quantitatively in terms of how the related optical image properties scale with distance. While image size for a fixed object scales directly with the inverse of its distance, its retinal disparity is inversely proportional to the square of the distance. As pointed out by Wallach and Zuckerman (1963) because of this difference, to achieve constancy these sources of information must be processed by different means.

See published version at https://doi.org/10.1016/j.visres.2020.10.003for

1.2 Stereoscopic Depth Constancy

Depth constancy is supported by multiple sources of static depth and distance information (e.g. relative size, perspective, accommodation, convergence and stereopsis). The known precision and utility of stereopsis in providing the 3D layout of surfaces and object shape suggests that stereopsis plays an important role in achieving depth constancy (Durgin et al., 1995; Frisby et al., 1996; McKee & Taylor, 2010). However, the focus of much of the stereoscopic depth constancy research has been on the source of egocentric distance signal and, relatedly, performance at distances that are within and outside personal space.

Distance scaling The question 'how do we know the distance of things' has been posed by philosophers and scientists over centuries and dates back (at least) to the work of Al-Hazen (Howard & Rogers, 2012). Subsequently, the possible sources of information or solutions were itemized, notably by Kepler in 1604 and Descartes in 1625 (as outlined in Sedgwick & Gillam, 2017). The role of stereopsis in depth scaling remained unrecognized until Wheatstone (1838) identified binocular disparity as source of depth information and went on to evaluate how it contributes to scene layout. With Wheatstone's discovery of the link between binocular disparity and depth perception, came the understanding that stereopsis could support depth constancy. Given that it was already widely believed that the convergence state of the eyes was the major source of visual information about distance, this oculomotor signal was the obvious candidate for scaling binocular disparity to achieve stereoscopic depth constancy. However, a number of psychophysical studies showed that (as suggested by Wheatstone) vergence on its own is not sufficient to support robust depth constancy (among others see Foley & Held, 1972; Gogel, 1961; Komoda & Ono, 1974).

Both Wheatstone (1838) and later Helmholtz (von Helmholtz, 1962) also noted that vertical disparities could be used to scale binocular disparities. The role of vertical disparity signals in binocular depth perception was highlighted by Ogle (1938) in his studies of the induced effect, and later by Howard (1970). Following the computational and geometric analyses of Longuet-Higgins and Mayhew (Longuet-Higgins, 1981; Mayhew & Longuet-Higgins, 1982) and Gillam and Lawergren (1983) the impact of vertical disparity signals on depth constancy received renewed psychophysical attention. A number of experiments have demonstrated that both vergence and vertical disparity can influence depth constancy (Banks et al., 2002; Foley et al., 1975; Rogers & Bradshaw, 1993). However, optimal scaling is typically only seen when both sources of information are available (Ritter, 1977; Rogers & Bradshaw, 1995; Swenson, 1932; Wallach & Zuckerman, 1963).

One of the most consistent results within this literature is that when near-veridical depth scaling is reported, distances tend to be less than 2 m (review by Ono & Comerford, 1977; Ritter, 1977; Wallach & Zuckerman, 1963). However, as outlined in Section 4 below, depth scaling does occur at much greater distances (Allison, Gillam, & Palmisano, 2009; Allison, Gillam, & Vecellio, 2009; Cormack, 1984; Palmisano et al., 2010). Another important but less recognized aspect of the stereoscopic depth constancy literature is the almost singular focus on predictions and performance for stimuli presented along the midline. However, as outlined in Section 2.3 below, binocular disparity varies with stimulus position i.e. head-centric direction. Because the horopter is curved, the relative disparity between two points changes substantially with eccentricity and under some conditions even reverses. As discussed below, it is not clear how this potentially important aspect of stereoscopic viewing geometry impacts depth constancy; few studies have addressed this issue directly. Our experience with the world suggests that the visual system is able to maintain constancy for objects and surfaces across a wide swath of visual space, not just along the midline. If so, the question that remains is how is this achieved and what other sources of depth information are used to maintain the apparent stability of visual space? Surprisingly little has changed in the 60 years since Ogle noted "The general problem of changes in the physiological and the optical processes of the eye in asymmetrical convergence is a complicated one, and more investigation is needed" (Ogle, 1962a, p. 343). At a minimum, we argue here that the apparent stability of perceived depth across changes in eccentricity is an impressive and generally under-appreciated achievement of the human visual system.

2 Geometry of stereoscopic depth constancy

2.1 Correspondence and the horopter

In order to understand the geometry of stereoscopic depth constancy it is important to consider the geometrical facts underlying binocular stereopsis. Details about the geometrical basis of stereopsis, binocular matching and physiological disparity processing are beyond the scope of the current paper (for review see Hartley & Zisserman, 2003; Howard & Rogers, 2012; Mayhew & Longuet-Higgins, 1982). It is enough for our purposes to note that the epipolar geometry combined with the current pose of the eyes constrains the 3D position of binocularly matched objects. The lateral separation of the two eyes in the head produces systematic differences in the direction of objects from these two vantage points. These disparities in the optic arrays are sufficient to localize the object's 3D position relative to the head. As we have mobile eyes, this epipolar geometry is not fixed relative to the retinas. Thus to determine the direction and distance of an object one must also account for eye position (e.g., Garding et al., 1995; Stevenson & Schor, 1997), either by monitoring extraretinal signals or by recovering the equivalent information from the retinal images (Banks & Backus, 1998; Longuet-Higgins, 1981).

The basic primitive of stereopsis is usually taken to be positional disparity, which reflects the depth of an object relative to the fixation point. For simplicity, here we assume spherical retinae centred on the nodal point of each eye and define corresponding points on the two retinae in terms of their geometric correspondence – that is as those points that have identical location (same spherical coordinates) relative to the fovea. For a more complete and nuanced discussion of horopters and corresponding points we refer the reader to standard texts on binocular vision (Howard & Rogers, 2012). If we precisely fixate an object its images will land on both fovea and we say that the object falls on corresponding points or has zero retinal disparity. The locus of object positions, including the fixation point, that project to corresponding points on the two retinae is called the horopter. In the absence of eye torsion, the horopter forms a circle through the fixation point and the nodal points of the two eyes (the Vieth-Müller circle, Figure 1) and a line perpendicular to and intersecting it in the median plane of the head (not shown in the figure).

The images of any object that does not lie on the geometrical horopter form on different locations on the left and right retinae. The difference in these retinal positions is the absolute retinal disparity (usually expressed in angular terms) which increases in magnitude with increased distance of the object from the horopter. Thus, the absolute disparity encodes distance from the horopter and its sign indicates its direction in depth (whether inside or outside the horopter). While a larger disparity indicates a greater separation from the horopter the relationship between the amount of depth and the magnitude of the disparity depends on the fixation distance. Furthermore, stereopsis is most sensitive to relative disparity, that is the difference in retinal disparity between one object and another (or between locations across a surface), which codes the relative distance or depth between the points. The relation between depth and relative disparity is not constant but depends on the location of the points in space.

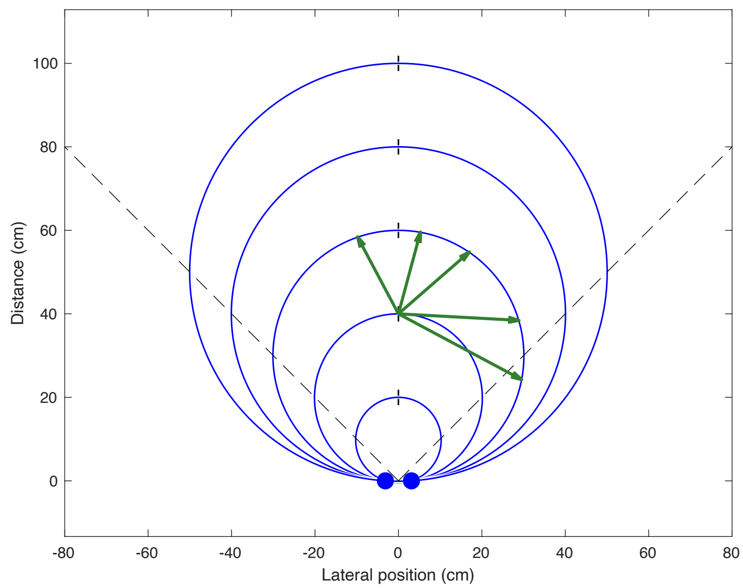

Figure 1: Vieth-Müller or equi-disparity circles at different fixation distances in the median plane (+ symbols) for a single interocular distance (blue filled circles indicate eye locations). For a given fixation distance other circles represent iso-disparity curves. The direction and size of the depth interval for a given disparity depends on where on the iso-disparity curves the points lie (arrows). The dashed lines show head centric direction of ±45° where the tangent to the horopter is orthogonal to the frontal plane.

The utility of the horopter relates to its definition of corresponding points and as a null disparity reference. The above definition of the horopter and its construction based on the Vieth-Müller circle and vertical line is entirely geometrical. The concept of the horopter underlying the Vieth-Müller circle involves equating visual angle on the two retinae which in turn implies equating the direction of visual lines relative to the fovea. A horopter measured by comparing perceived direction in the two eyes is known as a nonius horopter and is typically considered the most valid measure of the empirical horopter (Ogle, 1962a; Shipley et al., 1970). It is challenging to make these judgements at large eccentricities so related techniques like minimal apparent dichoptic motion are used (Ledgeway & Rogers, 1999; Nakayama, 1977; Schreiber et al., 2008). A classic finding is that the nonius horopter deviates in two significant ways from the theoretical geometrical horopter based on equating angles. First, the horizontal nonius horopter is less curved than Vieth-Müller circle (the Hering-Hillebrand deviation), as if points on the temporal hemi-retina are compressed relative to the corresponding points on the nasal retina of the other eye (Ames et al., 1932a; Schreiber et al., 2008; Shipley et al., 1970). Second, the vertical horopter is tilted backward, consistent with a shearing of corresponding points as originally suggested by Helmholtz (Ogle, 1950; Schreiber et al., 2008; von Helmholtz, 1962).

The horopter is important concept for stereopsis and is related to regions of best stereoacuity and binocular fusion but by its nature is a null or zero disparity curve. It does not speak directly to depth relative to the horopter but iso-disparity curves have the same form and the sign of disparity determines whether a point lies inside or outside the horopter. Performance on empirical tasks such as determining the apparent frontal plane and apparent equidistant surfaces can depend on depth constancy in at least two ways. First the observer must account for the curvature and slant of the horopter corresponding to the current vergence state of the eyes and second, depth scaling is needed to convert the relative disparities to depth (Garding et al., 1995). Note that the theoretical horopter is unchanged for fixation at any point on the Veith-Müller circle (i.e. with *asymmetric convergence)* but does change for the eyes in tertiary positions due to eye torsion (Schreiber et al., 2008; von Helmholtz, 1962).

2.2 Distance dependence

The relationship between depth (d) and disparity (η) depends on the fixation distance (D) and interocular distance (a). For objects near the midline and for depth much smaller than the fixation distance $\eta \approx \frac{ad}{D^2}$ (Cormack & Fox, 1985). This equation highlights the basic property needed for stereoscopic constancy, that is an inverse dependence of the disparity on the viewing distance squared (equivalently the depth predicted for a given disparity increases with the square of the distance, $d \approx \frac{\eta D^2}{a}$). Incorporation of this quadratic dependence on distance in calibrating depth from disparity would produce depth constancy. Because the relationship between angular size and linear size scales *linearly* with distance, to achieve stereoscopic shape constancy the size and depth need to be calibrated differently.

The other obvious parameter in the expression relating depth and disparity is the interocular distance, a, which is also required to reconstruct calibrated depth from disparity. While accounting for or scaling by a is needed for accurate depth reconstruction it not actually needed for depth constancy when the interocular distance is constant. As long as the distance squared relationship is accounted for, depth constancy will hold as depths will be invariant with distance although inaccurate if improperly scaled. However, the nodal points of the eye do not coincide with the eye's centre of rotation and thus perfect depth constancy would also need to account for the reduction in effective a due to convergence when viewing distances are very close (Mapp & Ono, 1986)[1].

Most real-life objects are comprised of surfaces rather than just isolated points and edges. Patterns of disparity over a spatially extended surface indicate surface depth, slant, curvature and other aspects of shape. Several authors have noted that surface slant is specified by a gradient of disparity (for review see Howard & Rogers, 2012). As the disparity gradient depends on both the

[1] Note that while the shape of the horopter for a given fixation distance is similar for different interocular distances (except at large eccentricity), the amount of vergence required differs.

disparity between points on a slanted surface and their separation, the theoretical slant corresponding to a given disparity gradient increases linearly with distance rather than with the square of distance. Depth curvature is specified by the second derivative of disparity and thus should be invariant with distance (Rogers & Cagenello, 1989). These considerations indicate that distance scaling for stereoscopic slant judgments must compensate for distance while depth curvature judgements should in theory be invariant and need no compensation to exhibit constancy. However, the practicality of this invariant has been questioned since it only holds for objects near the median plane of the head and for local curvature, not for overall shape (Howard & Rogers, 2012). Another interesting invariant was identified by Rogers and Bradshaw (Rogers & Bradshaw, 1993) who showed that a frontal surface is uniquely signalled when the dichoptic ratio of horizontal sizes of small texture elements on a surface (or between features) equals the square of their vertical size ratio ($HSR = VSR^2$) at all points along the surface. Vertical size ratios are a measure of vertical disparity, its role in supporting stereoscopic depth constancy has been indicated by a number of theoretical and experimental studies (e.g., Backus et al., 1999; Gillam & Lawergren, 1983; Howard & Kaneko, 1994; Mayhew & Longuet-Higgins, 1982; Ogle, 1938; Rogers & Bradshaw, 1993; von Helmholtz, 1962).

2.3 Direction dependence

The focus of most depth constancy research has been on stimuli presented directly along the midline. However, disparity also varies substantially with the azimuth direction (head-centric direction or eccentricity relative to the cyclopean eye) of a stimulus and thus direction is also important for constancy[2]. The theoretical horizontal point horopter (the Vieth-Müller circle) always passes through the fixation point and the nodal points of the two eyes, therefore its radius decreases with convergence of the eyes (Figure 1). Thus, the horopter curves for different fixation distances converge as they approach the nodal points of the two eyes. Equivalently, iso-disparity curves for a given fixation distance also converge at the nodal points. As a result, the geometrically predicted depth for a given disparity decreases as its azimuth direction along the horopter increases, irrespective of gaze position. The theoretical horopter curves strongly inward toward the observer and thus a horopter (or iso-disparity curve) does not correspond to a constant egocentric or frontal distance.

Consider a thin rod oriented perpendicular to a frontal plane at a given distance (Figure 2a). The relative disparity between the front and the back of the object varies as a function of its azimuth direction in a counterintuitive fashion. At any given distance, as the object is translated further to the left or right across the plane, the disparity decreases and eventually becomes negative (the disparity of the 'near' part becomes less than the far part, Figure 2b). Plotting the relationship in angular terms shows that this reversal occurs at approximately 45º azimuth (Figure 2c). This is the point at which the tangent to the Vieth-Müller circle is perpendicular to the frontal plane and the horopter begins to curve back toward the midline (Figure 1). Thus the 'near' end of the rod

[2] One reason for this dependence is that the effective separation between the nodal points of the eyes (stereoscopic baseline) decreases with the azimuth direction of a target (head-centric eccentricity). This can easily be seen by considering the angle subtended by this baseline at various points along an equidistance circle centred on the cyclopean eye. The subtended angle is maximum when the point is straight-ahead and decreases as the cosine of the azimuth, becoming zero when at 90º when the nodal points of the eyes and the point are collinear.

now lies outside rather than inside the iso-disparity curve[3] passing through the back end. Therefore, as an object moves in a frontal plane the disparity within it varies and can even reverse sign. Given that in our everyday experience eccentrically located objects do not appear attenuated or inverted in depth, the visual system must have depth constancy for these viewing conditions.

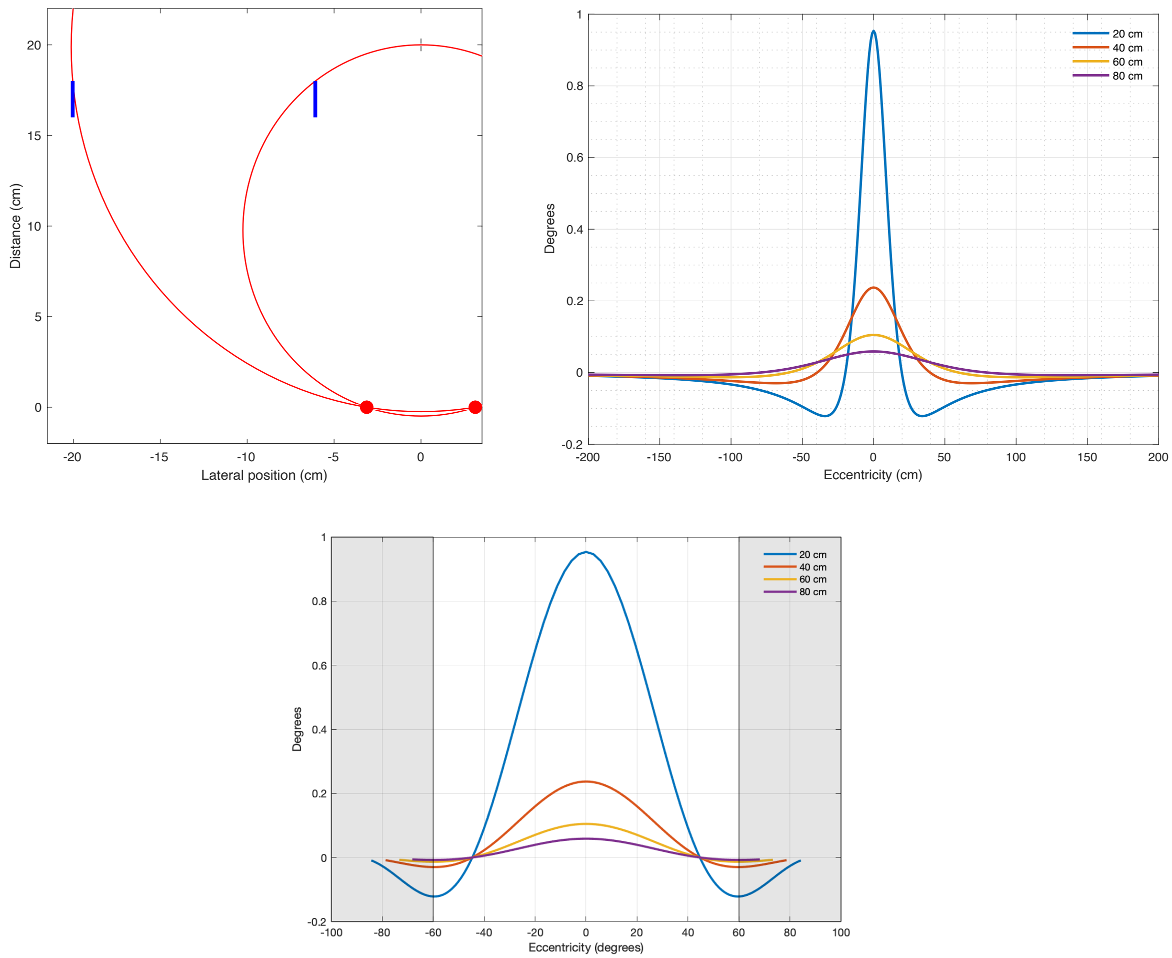

Figure 2 (a) Thin rods oriented perpendicular to the frontal plane (blue lines) with their far end at a distance of 18 cm and laterally displaced a small amount (6 cm) or a large amount (20 cm) to the left relative to the eyes with an IPD of 6.25 cm (small filled circles). The nearer end lies inside the respective Vieth-Müller circle for fixation on the back of the rod (red circles) when centrally located but outside when laterally placed. (b,c) The relative disparity between the near and far end of a 1 cm rod at various frontal plane distances (colours) is show as a function of eccentricity in linear (b) or angular (c) units. Gray shading in (c) shows the approximate region of the visual field visible to only one eye (maximum extent of the nasal visual field) and thus where disparity is not available (Glaser, 1967).

2.4 Independence of distance and direction as determinants of disparity

Distance and depth might be more naturally represented in terms of egocentric distance radially out from the observer rather than perpendicular distance from the frontal plane of the head. If the targets are separated along a radial line from the cyclopean eye (midpoint between the two eyes) then the reversal of disparity with direction does not occur as the radially nearer inner end always

[3] This would be the horopter if the object were accurately binocularly fixated.

lies inside the iso-disparity circle containing the outer end. However, while the sign of the disparity is constant there is a dependence on both distance and direction at larger eccentricities (see Appendix).

The geometrical dependence of disparity on distance implied by the inverse square law (Figure 3) is remarkably independent of direction when distance is expressed as distance to a frontal plane containing the reference point (distance in the z-direction). This independence presumably simplifies the neural processing underlying depth constancy because distance and direction can be accounted for separately. For example, the brain could learn a canonical relationship between disparity and direction and then scale this according to distance (as was done in Figure 3).

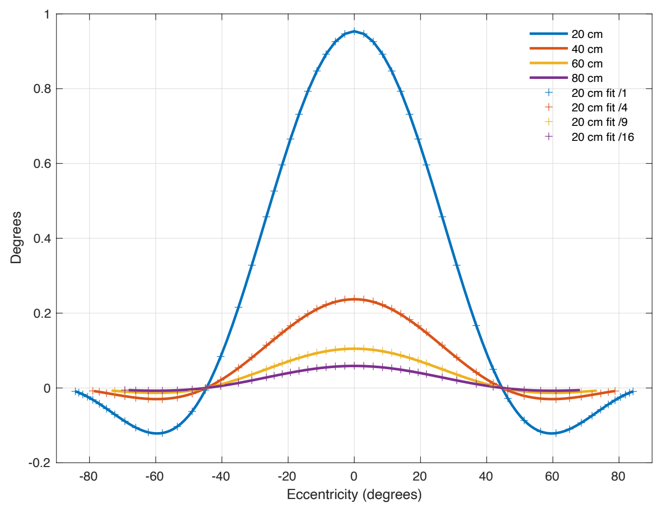

Figure 3 Disparity for a 1 cm depth interval (perpendicular to the frontal plane) as function of distance and direction (solid lines) compared with an approximation by $1/D^2$ scaling of the 20 cm curve to other distances (+ symbols).

3 Behavioral evidence of stereoscopic depth constancy

Strong evidence for depth constancy is obtained when observers' depth estimates are veridical at multiple viewing distances or directions. However, such accuracy is not required for constancy as long as perceived depth is invariant with location in space. Review of the psychophysical literature on depth constancy shows that some degree of constancy is typically seen at relatively short viewing distances (less than 2 m) for stimuli presented along the midline. This work is reviewed in detail by Ono and Comerford (1977) and by Foley (1980) so an overview is provided here.

3.1 Evidence for distance scaling

Wallach and Zuckerman (1963) published one of the first series of experiments aimed directly at assessing depth constancy and the factors that contribute to it. They asked observers to estimate either the height (depth) or width (size) of a wire-frame pyramid presented at 66.5 and 133 cm. They showed that perceived depth was close to veridical (with slight overestimates), even though only convergence and accommodation were available to provide egocentric distance information. They concluded that our percept of the size of a depth interval must take into account the inverse square relationship between disparity and distance. The near-veridical depth constancy reported by Wallach and colleagues at near viewing distances has been seen in other evaluations. For example, Ritter (1977) used a similar methodology and found nearly perfect depth constancy at near viewing distances under natural viewing conditions which included motion parallax. The absence of depth cue conflicts in real world stimuli is likely one of the main reasons for the more complete depth scaling found for such stimuli compared to those presented virtually (at near distances). Interestingly, Ritter (1977) also reported that removing the motion information had no discernable impact on depth constancy, nor did limiting accommodation (a result also reported by Swenson (1932)).

It is evident from the experiments highlighted above that depth constancy is seen in near space when viewing stimuli that have multiple consistent depth cues, as is the case with physical arrangements. Much of the literature on depth constancy has attempted to identify what sources of information provide the requisite estimate of egocentric distance.[4] A detailed examination of these is beyond the scope of this paper, and there is considerable variability in the outcomes. However, in general it is clear that both vergence and vertical disparity play an important role, although neither vergence (Foley & Held, 1972; Gogel, 1961; Komoda & Ono, 1974) or vertical disparity are sufficient on their own (Banks et al., 2002; Foley et al., 1975; Rogers & Bradshaw, 1993). Experiments that have used real objects have tended to find more accurate distance estimation, for instance via vergence (Durgin et al., 1995; Mon-Williams et al., 2000). Other factors that contributed to the completeness of depth constancy include whether observers have the opportunity to make multiple vergence eye movements and the measurement method (Foley & Held, 1972; Glennerster et al., 1996). In the latter case for example, it appears that perceptual estimation methods like those used by Wallach and colleagues, Ritter (1977) and Gogel (1961) produce more accurate depth scaling, while manual response methods like that used by Foley and his collaborators result in over-estimation.

Another important topic evaluated by Wallach and Zuckerman (1963) is the contribution of perspective information to depth constancy. To assess this, they used a pseudoscope to present pairs of anaglyphs at different distances either floating in space, or on a surface with strong perspective information. They found that the perspective cues overrode the binocular cues in determining perceived depth. This study was performed at relatively close distances of 3 and 5 feet, beyond these distances it is often assumed that stereoscopic depth constancy fails. However relatively little evaluation of depth magnitude perception has been performed at longer distances, particularly under natural viewing conditions. Cormack (1984) demonstrated that depth could be matched using a disparity probe at distances up to 17.8 km. He also reported that egocentric distance estimates for stereoscopic afterimages appeared close to their predicted value for fixation distances up to 20 m. More recently, Allison, Gillam, and Vecellio (2009) and Palmisano, Gillam, Govan, Allison and Harris (2010) showed that binocular depth estimation scales (albeit incompletely) at reference distances of 4.5 to 18 m and 20 to 40 m, respectively. As

[4] It has also been proposed that distance estimation and stereoscopic depth estimation are separate processes and that other visual invariants are used to scale perceived depth (Epstein, 1973, 1995; Rogers & Cagenello, 1989; Sedgwick & Gillam, 2017; Vreven, 2006).

in Cormack's study, these distances are outside the useful range of accommodation, vergence and vertical disparity. Taken together, their results underscore the potential impact of monocular perspective cues on stereoscopic depth scaling. Others have shown that in addition to monocular information about distance, monocular size and depth information influence stereoscopic depth constancy (Brenner & van Damme, 1999; Collett et al., 1991; Foley, 1968; Mon-Williams et al., 2000; O'leary & Wallach, 1980).

It is worth restating here that while there are many manipulations that impact depth constancy, there is an abundance of evidence showing that given sufficient information concerning egocentric distance, binocular stereopsis does indeed support consistent perception of depth intervals up to distances of 2 m. At larger distances depth scaling is incomplete but it is clear that, at least to 40 m, stereoscopic depth constancy does occur.

3.2 Evidence for eccentricity scaling

As noted above stereopsis with laterally displaced objects and surfaces must contend with (1) a rotation of the stereoscopic reference frame and (2) recalibrating the relationship between depth and disparity to reflect the reduction in effective stereoscopic baseline. The first concern recognizes that the theoretical geometric horopter aligns with neither a frontal surface nor an equidistant surface, both of which would seem to form intuitive references for stereopsis. This has long been recognized as an important theoretical problem in binocular vision. Few studies have looked at the second question of depth constancy away from the midline although several have looked at the empirical equidistant or apparent frontal horopters with asymmetrical convergence, which can be considered specialized cases of depth constancy.

Amigo (1965, 1972) considered the effect of asymmetric convergence (0, 10, 20 and 30º) on stereoscopic depth. He attempted to measure 'the stereoscopic reference curves' – locus of points that gave rise to a sense of being 'equidistant' to the fixation point – in the presence or absence of contours providing vertical disparity (Amigo, 1972). This has sometimes been called the equidistance horopter. The observers were instructed to adjust test stimuli to appear equidistant to the fixation stimulus; however, the definition of equidistant was somewhat vague. From his figures it seems that what was meant was lying on a plane through the fixation point and normal to the cyclopean line of sight at the fixation point. If veridical, this plane would have slant equal to the gaze azimuth and would be ½ of the local slant of the horopter (Gillam & Lawergren, 1983; Miller & Ogle, 1964 but see Morrison 1977). Interestingly, the curves Amigo obtained were closer to the objective normal plane in the presence of vertical disparity and with increasing distance. Thus, although there were substantial inter-observer differences there appeared to be some evidence of compensation for eccentric fixation (asymmetric convergence).

Other experimenters have used the criteria of equidistance to the cyclopean eye to measure this apparent equidistant locus (Foley, 1966, 1970), although Howard & Rogers (Howard & Rogers, 2012) have noted that the task might be unclear to some observers. For instance, Foley (1966) asked observers to match the radial distance of targets at azimuth directions of up to 24º to a central standard at one of four distances (1.2–4.2 m). As in Amigo's (1972) results there was considerable inter-subject variability in asymmetry and curvature, but Foley found that the perceptually equidistant locus was always more concave than the actual equidistant circle (but not as strongly curved as the Vieth-Müller circle). The disparities of the matched points relative to the Vieth-Müller circle increased as distance decreased (Foley, 1966, 1970). Thus, his results provide some evidence for constancy, but it was imperfect. This is likely due to the fact that his targets were small lights located along the horizon plane so they provided no vertical disparity information which has been shown to improve constancy for the apparent frontal plane task.

Vertical disparities increase in a nearly linear fashion with eccentricity and the magnitude of the gradient increases with nearness (inverse distance) (Gillam & Lawergren, 1983; Mayhew & Longuet-Higgins, 1982). Brenner et al (Brenner et al., 2001) argued that if the vertical size difference between the left and right images was used directly for distance scaling in a given direction then constancy should improve with increasing azimuth (lateral position) of the target. This is because the vertical size ratio varies more with distance at greater head-centric eccentricity (the images of objects in the mid-sagittal plane are equal vertical size at all distances). In their study, participants adjusted the size and shape of a textured ellipsoid presented on a stereoscopic display to match those of a tennis ball. From these matches they estimated the 'size scaling' distance at which the adjusted retinal image size would correspond to the projection of a tennis ball[5]; similarly, they calculated the 'shape scaling' distance where the ratio of size to disparity would correspond to a sphere. They hypothesized that the compression of the scaling distances (toward the actual screen distance) that they found for targets presented straight-ahead would not be seen when the head was rotated 30º in azimuth relative to the screen. However, contrary to this hypothesis, the slope between scaling distance and simulated distance did not differ between the two viewing conditions. Combined with their finding of improved distance scaling with larger display extents, they concluded that the visual systems uses gradients of vertical size disparity (rather than vertical size disparity itself) to scale depth for distance. They further suggested that the visual system used these gradients to estimate egocentric distance. Participants did not estimate distance in these experiments so this assumption was not tested[5]. Note that while the slope between scaling distance and simulated distance did not differ between viewing conditions the 'shape scaling' distances were smaller in the 30º than 0º conditions (from the fitted lines in their Figure 3D the ratio ranged from 0.84 to 0.93 with an average ratio of 0.89; they did not comment on this or report any tests of this difference). The authors noted that when the head was turned relative to the display that they 'rendered the images in accordance with the asymmetric eye positions'. We would expect the disparity for displaced settings to be smaller than those made centrally (disparity at 30º would be approximately 0.87 times the disparity of the same target straight-ahead).

These studies have looked at the scaling of disparity in perception but we would expect constancies to also be reflected in visually-guided action. Greenwald and Knill (Greenwald & Knill, 2009) measured both slant discrimination thresholds and grasping for targets at various retinal eccentricities and disparity relative to the horopter. They found that stereopsis was relied on less with increasing eccentricity and disparity a result which they attributed to its reduced reliability under these conditions. These factors depend on the retinal location and thus utility of stereopsis should be restored if the target was fixated. Normally when one needs to interact precisely with an object it is also fixated. However, the dependence on retinal eccentricity would be functionally important for the perception of an extended surface, for motor planning and for

[5] They implicitly assumed the size-distance invariance hypothesis in both the scaling distance analysis and the interpretation of their results.

interaction with competing attentional demands. The study is notable in using the horopter as the binocular frame of reference in the analysis, which is uncommon in studies of binocular control of prehension.

Given the evidence that head-centric direction seems to be accounted for when making stereoscopic depth judgements we might ask how this information is obtained. With a mobile eye, visual direction cannot be obtained from retinal position alone and the stability and accuracy of visual direction is subject to its own type of constancy called direction constancy (Hill, 1972; Morgan, 1978). Extraretinal information about eye position can be obtained from eye proprioception or efference copy and spatial updating (Crawford et al., 2011). Ebenholtz and Paap (Ebenholtz & Paap, 1976) looked at the effects of sensed eye position by adapting observers' perceived straight-ahead by prism adaptation or prolonged eccentric gaze. They found that observers exhibited biases in perceived slant in a direction consistent with the biases in perceived gaze direction. Alternatively, direction information could be obtained from the retinal images, in particular from vertical size disparity. In his investigations of the impact of differential meridonial magnification in the two eyes Ogle (1938) showed that magnification of one eye's image along the vertical meridian induces a perception of slant that is consistent with, but the reverse of, the same degree of horizontal magnification in the other eye. Such vertical size disparities naturally arise when viewing an eccentric surface and several investigators have suggested this relationship could be turned around and vertical size disparity could be used as an indicator of gaze direction. Berends et al (Berends et al., 2002) measured perceived straight-ahead and found, consistent with other reports (Banks et al., 2002), that vertical disparity did not directly influence visual direction. However, there was evidence for its role in calibration as 5-min adaptation to vertical size disparity shifted perceived visual direction in 5 of 9 participants. The shifts in perceived straight-ahead were small relative to the predictions from a model of vertical size disparity directly indicating gaze eccentricity (Householder, 1943; Mayhew & Longuet-Higgins, 1982). Thus, it seems that vertical disparity is used in the direction dependent calibration of stereopsis but not in the estimation of direction.

4 Empirical relationship between the horopter and depth constancy

Despite the mathematical nature and practical difficulties in operationally defining and measuring the horopter, the concept—particularly of the longitudinal horopter—is key to defining the frame of reference for stereopsis (Ogle, 1962b). The suprathreshold depth associated with depth constancy builds around and reflects the shape, slant, and spacing of the horopter curve. As discussed in section 2.1 the theoretical horopter is defined as the "the locus of single points in space, each of which projects images onto corresponding points in the two retinas" (Howard & Rogers, 2012, p. 38). A wide range of psychophysical techniques have been used to measure empirical horopters, many of these use dichoptic techniques (e.g. the nonious grid method described by Ames et al., 1932a, 1932b) which permit equating of visual directions (and therefore identifying corresponding points). Hering had originally argued that if points were positioned on an apparently frontal plane (AFP) they would stimulate corresponding points (described by Foley, 1980). Although this is not the case, the AFP has since received considerable empirical attention. The frontal plane task (Ames et al., 1932a; Ogle, 1950; von Helmholtz, 1962) is more stable and reliable than nonius tasks; however, because disparities along a surface change with viewing distance it cannot be based on the pattern of disparity alone (Howard & Rogers, 2012). Thus we concur with earlier authors that the AFP is not a measure of the horopter at all but instead measures and reflects depth constancy (Ogle, 1950; Shipley et al., 1970). Ogle (1950) reported that when gradients of disparity were introduced by magnification the resulting slanted surfaces showed the same Hering-Hillebrand deviation as the AFP (in addition to the slant) further suggesting that the horopter geometry is reflected in processing depth from disparity.

When the eyes are asymmetrically converged to fixate a point on the horopter away from the midline[6] the pattern of disparities that specified a frontal plane at the midline now specify a surface tangent to the horopter. Ebenholtz and Paap (Ebenholtz & Paap, 1973) are the most direct in linking compensation for the shape of the horopter with depth constancy. They noted that a slanted central surface can have the same disparity pattern as a frontal surface patch displaced to one side of straight-ahead. The tangent plane to the Veith-Müller circle (which they called the reference surface for depth perception) is increasingly slanted away from the frontal plane with head-centric eccentricity (Figure 1); perception of surface slant would need to compensate for this rotation to achieve slant constancy (the rotation is approximately equal to the gaze azimuth to fixate the surface (Ogle, 1950)). To determine whether the visual system performs this compensation, they had observers adjust the slant of a central comparison line to match the slant of (1) a thin vertical line presented either centrally or displaced above or below straight-ahead and (2) a horizontal line presented either centrally or displaced to the left or right of straight-ahead. Observers could fixate the test lines which were either frontal or slanted relative to the frontal plane. They found that slant matches were accurate for vertical displacements and nearly veridical for lateral displacements. Thus, participants were able to compensate for the slant of the geometrical horopter in asymmetric convergence to maintain constancy. The errors seen in lateral displacement conditions were consistent with underestimation of direction, however, pointing measures of perceived direction did not corroborate this explanation. Ebenholtz & Paap (1973) explicitly linked this slant constancy to measurements of the horopter concluding ‘The joint processing of retinal disparity and displacement angle is presumed to underlie orientation constancy, as exhibited under the circumstances of the present investigation. It is likely that the same interpretation holds for the horopter studies of the apparent frontoparallel plane in asymmetrical convergence’. These results, along with findings that obtained settings for the normal plane are rotated with respect to the nonius horopter (Ogle, 1950) or Veith-Müller circle (Amigo, 1972; Backus et al., 1999; Ogle, 1950) indicates constancy for direction at least in compensating for the slant of the horopter.

5 Conclusion

The analysis presented here highlights the both the complexity of binocular viewing geometry and the extent to which the visual system compensates for this to maintain depth constancy. The primary focus on depth constancy along the midline has left an impressive degree of depth constancy under-recognized and under-explored and obscured the relationship between depth constancy and the horopter (including the geometric and induced effects). Guan and Banks

[6] The geometry is the same for symmetric fixation at the midline and an eccentric surface at another point on the horopter although judgements would typically be more difficult in this case.

(2016) have recently made similar arguments for the consideration of scale as another important factor for depth constancy. A more general and useful framework for stereoscopic depth constancy must take into account not just distance but egocentric location in space and modulating factors such as spatial scale. While the experiments cited above suggest that depth constancy for direction occurs, more work is needed to assess its completeness, the extent to which it relies on different sources of distance information, and how it varies with viewing distance.

Further, it is arguable that failure to recognize the importance of considering the shape of the horopter in depth constancy can lead to incorrect predictions for perceived depth or shape under stereoscopic viewing. This has a number of potential implications for modern technologies. For instance, in augmented reality 3D mapping of visual space and placement of objects within that space requires a high degree of accuracy as users can reference simulated object shape and position with the structure of the real world. Our analysis and observations suggest that the human visual system readily compensates for geometric distortions that occur for eccentric objects. However, we predict that mis-registration of distance or direction by the display device will result in substantial perceptual distortion. In addition to technical issues in registering distance or direction, anatomical variation (IPD, eye relief) and variation in fit can make it difficult to precisely specify 'where things are coming from' in a head-mounted display (HMD). Furthermore, to this point we have focussed solely on the implications for the *perception* of depth. In both real and virtual environments we interact with and locomote through environments, and often interact with objects that are positioned eccentrically. It is important to also consider the implications of this analysis for prehension, and the nature of depth constancy when acting upon the world.

6 Appendix

Distance and depth might be more naturally represented in terms of egocentric distance radially out from the observer rather than perpendicular distance from the frontal plane of the head (Figure 4a). If the targets are separated along a radial line from the cyclopean eye (midpoint between the two eyes) then the reversal of disparity with direction does not occur as the radially nearer inner end always lies inside the iso-disparity circle containing the outer end. Figure 4b shows the geometric disparity for a radial depth of 1 cm relative to equidistant circles at various distances. While the sign of the disparity is constant there is a dependence on both distance and direction at larger eccentricities.

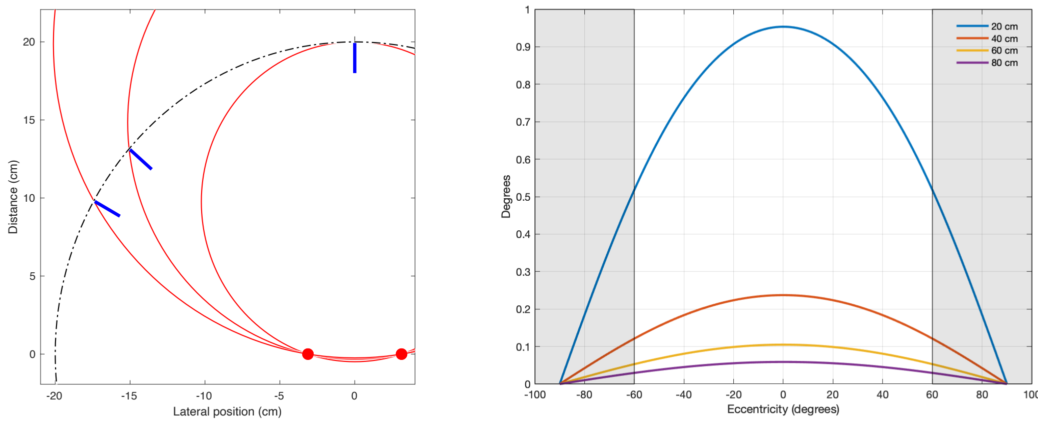

Figure 4 (a) Thin rods (blue) oriented perpendicular to a 20-cm radius equidistant cylinder (black line) centred on the 'cyclopean eye' located midway between the two eyes, shown as small red dots). The nearer end lies inside the respective Vieth-Müller circle for fixation on the back of the rod (red circles) for both centrally located and laterally placed rods. (b) Relative disparity for a 1 cm thin rod target aligned along various cyclopean directions as function of the direction and the radial distance of an equidistant surface at the back of the object. Gray shading in (b) shows the approximate region of the visual field visible to only one eye (maximum extent of the nasal visual field) and thus where disparity is not available (Glaser, 1967).

As shown in Figure 3 the dependence of disparity with the inverse of distance squared is remarkably independent of direction when distance is expressed as distance to a frontal plane containing the reference point. When distance is expressed as radial distance from the cyclopean eye the approximation is still very good but has increased error for eccentric visual directions (not shown).

Acknowledgements

The authors would like to gratefully acknowledge the financial support of NSERC Canada through Discovery Grants RGPIN-2020-06061 (RA) and RGPIN-2019-06694 (LW). This research was undertaken as part of the Vision: Science to Applications program, thanks in part to funding from the Canada First Research Excellence Fund.

References

Allison, R. S., Gillam, B. J., & Palmisano, S. A. (2009). Stereoscopic discrimination of the layout of ground surfaces. *Journal of Vision*, *9*(12 Article 8), 1–11. https://doi.org/10.1167/9.12.8

Allison, R. S., Gillam, B. J., & Vecellio, E. (2009). Binocular depth discrimination and estimation beyond interaction space. *Journal of Vision*, *9*(1 Article 10), 1–14. https://doi.org/10.1167/9.1.10

Ames, A., Ogle, K. N., & Gliddon, G. H. (1932a). Corresponding retinal points, the horopter and size and shape of ocular images, Part II. *J Opt Soc Am*, *22*(11), 575–631.

Ames, A., Ogle, K. N., & Gliddon, G. H. (1932b). Corresponding retinal points, the horopter and size and shape of ocular images, Part I. *JOSA*, *22*(10), 538–574. https://doi.org/10.1364/JOSA.22.000538

Amigo, G. (1965). The mathematical treatment of horopter data obtained with the eyes in asymmetric convergence. *Optica Acta: International Journal of Optics*, *12*(3), 305–315. https://doi.org/10.1080/713817940

Amigo, G. (1972). The stereoscopic frame of reference in asymmetric convergence of the eyes. *Optica Acta: International Journal of Optics*, *19*(12), 993–1006. https://doi.org/10.1080/713818524

Backus, B. T., Banks, M. S., van Ee, R., & Crowell, J. A. (1999). Horizontal and vertical disparity, eye position, and stereoscopic slant perception. *Vision Research*, *39*(6), 1143–1170. https://doi.org/10.1016/S0042-6989(98)00139-4

Banks, M. S., & Backus, B. T. (1998). Extra-retinal and perspective cues cause the small range of the induced effect. *Vision Research*, *38*(2), 187–194. https://doi.org/10.1016/S0042-6989(97)00179-X

Banks, M. S., Backus, B. T., & Banks, R. S. (2002). Is vertical disparity used to determine azimuth? *Vision Research*, *42*(7), 801–807. https://doi.org/10.1016/S0042-6989(01)00283-8

Berends, E. M., van Ee, R., & Erkelens, C. J. (2002). Vertical disparity can alter perceived direction. *Perception*, *31*(11), 1323–1333. https://doi.org/10.1068/p3440

Brenner, E., Smeets, J. B. J., & Landy, M. S. (2001). How vertical disparities assist judgements of distance. *Vision Research*, *41*(25), 3455–3465. https://doi.org/10.1016/S0042-6989(01)00206-1

Brenner, E., & van Damme, W. J. M. (1999). Perceived distance, shape and size. *Vision Research*, *39*(5), 975–986. https://doi.org/10.1016/S0042-6989(98)00162-X

Collett, T. S., Schwarz, U., & Sobel, E. C. (1991). The interaction of oculomotor cues and stimulus size in stereoscopic depth constancy. *Perception*, *20*(6), 733–754. https://doi.org/10.1068/p200733

Cormack, R. (1984). Stereoscopic depth perception at far viewing distances. *Perception & Psychophysics*, *35*, 423–428. https://doi.org/10.3758/BF03203918

Cormack, R., & Fox, R. (1985). The computation of retinal disparity. *Perception & Psychophysics*, *37*(2), 176–178. https://doi.org/10.3758/BF03202855

Crawford, J. D., Henriques, D. Y. P., & Medendorp, W. P. (2011). Three-dimensional transformations for goal-directed action. *Annual Review of Neuroscience*, *34*, 309–331. https://doi.org/10.1146/annurev-neuro-061010-113749

Durgin, F. H., Proffitt, D. R., Olson, T. J., & Reinke, K. S. (1995). Comparing depth from motion with depth from binocular disparity. *Journal of Experimental Psychology: Human Perception and Performance*, *21*(3), 679–699. https://doi.org/10.1037/0096-1523.21.3.679

Ebenholtz, S. M., & Paap, K. R. (1973). The constancy of object orientation: Compensation for ocular rotation. *Perception & Psychophysics*, *14*(3), 458–470. https://doi.org/10.3758/BF03211184

Ebenholtz, S. M., & Paap, K. R. (1976). Further evidence for an orientation constancy based upon registration of ocular position. *Psychological Research*, *38*(4), 395–409. https://doi.org/10.1007/BF00309044

Epstein, W. (1973). The process of 'taking-into-account' in visual perception. *Perception*, *2*(3), 267–285. https://doi.org/10.1068/p020267

Epstein, W. (1995). The Metatheoretical Context. In W. Epstein & S. Rogers (Eds.), *Perception of Space and Motion* (pp. 1–22). Academic Press.

Foley, J. M. (1966). Locus of Perceived Equidistance as a Function of Viewing Distance. *Journal of the Optical Society of America*, *56*(6), 822-. https://doi.org/10.1364/JOSA.56.000822

Foley, J. M. (1968). Depth size and distance in stereoscopic vision. *Perception & Psychophysics*, *3*(4A), 265-. https://doi.org/10.3758/BF03212742

Foley, J. M. (1970). Loci of perceived, equi-, half- and double-distance in stereoscopic vision. *Vision Research*, *10*(11), 1201–1209. https://doi.org/10.1016/0042-6989(70)90037-4

Foley, J. M. (1980). Binocular distance perception. *Psychological Review*, *87*(5), 411–434. https://doi.org/10.1037/0033-295X.87.5.411

Foley, J. M., Applebaum, T. H., & Richards, W. A. (1975). Stereopsis with large disparities: Discrimination and depth magnitude. *Vision Research*, *15*(3), 417–421. https://doi.org/10.1016/0042-6989(75)90091-7

Foley, J. M., & Held, R. (1972). Visually directed pointing as a function of target distance, direction, and available cues. *Perception & Psychophysics*, *12*(3), 263-. https://doi.org/10.3758/BF03207201

Frisby, J. P., Buckley, D., & Duke, P. A. (1996). Evidence for Good Recovery of Lengths of Real Objects Seen with Natural Stereo Viewing. *Perception*, *25*(2), 129–154. https://doi.org/10.1068/p250129

Garding, J., Porrill, J., Mayhew, J. E. W., & Frisby, J. P. (1995). Stereopsis, vertical disparity and relief transformations. *Vision Research*, *35*(5), 703–722. https://doi.org/10.1016/0042-6989(94)00162-F

Gillam, B. (2000). Perceptual constancies. In A. E. Kazdin (Ed.), *Encyclopedia of psychology, Vol. 6.* (pp. 89–93). American Psychological Association. https://doi.org/10.1037/10521-032

Gillam, B., & Lawergren, B. (1983). The induced effect, vertical disparity, and stereoscopic theory. *Perception & Psychophysics*, *34*(2), 121–130. https://doi.org/10.3758/BF03211336

Glaser, J. S. (1967). The nasal visual field. *Archives of Ophthalmology*, *77*(3), 358–360. https://doi.org/10.1001/archopht.1967.00980020360013

Glennerster, A., Rogers, B. J., & Bradshaw, M. F. (1996). Stereoscopic depth constancy depends on the subject's task. *Vision Research*, *36*(21), 3441–3456. https://doi.org/10.1016/0042-6989(96)00090-9

Gogel, W. C. (1961). Convergence as a cue to absolute distance. *The Journal of Psychology*, *52*(2), 287–301. https://doi.org/10.1080/00223980.1961.9916529

Greenwald, H. S., & Knill, D. C. (2009). Cue integration outside central fixation: A study of grasping in depth. *Journal of Vision*, *9*(2), 11–11. https://doi.org/10.1167/9.2.11

Guan, P., & Banks, M. S. (2016). Stereoscopic depth constancy. *Phil. Trans. R. Soc. B*, *371*(1697), 20150253. https://doi.org/10.1098/rstb.2015.0253

Hartley, R., & Zisserman, A. (2003). *Multiple view geometry in computer vision* (2nd ed.). Cambridge University Press.

Hill, A. L. (1972). Direction constancy. *Perception & Psychophysics*, *11*(2), 175–178. https://doi.org/10.3758/BF03210370

Householder, A. S. (1943). A theory of the induced size effect. *The Bulletin of Mathematical Biophysics*, *5*(4), 155–160. https://doi.org/10.1007/BF02478263

Howard, I. P. (1970). Vergence, eye signature, and stereopsis. *Psychonomic Monograph Supplements*, *3*, 201–204.

Howard, I. P., & Kaneko, H. (1994). Relative shear disparities and the perception of surface inclination. *Vision Research*, *34*(19), 2505–2517. https://doi.org/10.1016/0042-6989(94)90237-2

Howard, I. P., & Rogers, B. J. (2012). *Perceiving in depth, Volume 2 Stereoscopic vision: Vol. 2 Stereoscopic Vision*. Oxford University Press.

Komoda, M. K., & Ono, H. (1974). Oculomotor adjustments and size-distance perception. *Perception & Psychophysics*, *15*(2), 353–360. https://doi.org/10.3758/BF03213958

Ledgeway, T., & Rogers, B. J. (1999). The effects of eccentricity and vergence angle upon the relative tilt of corresponding vertical and horizontal meridia revealed using the minimum motion paradigm. *Perception*, *28*(2), 143–153. https://doi.org/10.1068/p2738

Longuet-Higgins, H. C. (1981). A computer algorithm for reconstructing a scene from two projections. *Nature*, *293*(5828), 133–135. https://doi.org/10.1038/293133a0

Mapp, A. P., & Ono, H. (1986). The rhino-optical phenomenon: Ocular parallax and the visible field beyond the nose. *Vision Research*, *26*(7), 1163–1165. https://doi.org/10.1016/0042-6989(86)90050-7

Mayhew, J. E. W., & Longuet-Higgins, H. C. (1982). A computational model of binocular depth perception. *Nature*, *297*(5865), 376–378. https://doi.org/10.1038/297376a0

McKee, S. P., & Taylor, D. G. (2010). The precision of binocular and monocular depth judgments in natural settings. *Journal of Vision*, *10*(10), 5–5. https://doi.org/10.1167/10.10.5

Miller, T. J., & Ogle, K. N. (1964). Stereoscopic localization of afterimages with eyes in asymmetric convergence. *Investigative Ophthalmology & Visual Science*, *3*(3), 339–353.

Mon-Williams, M., Tresilian, J. R., & Roberts, A. (2000). Vergence provides veridical depth perception from horizontal retinal image disparities. *Experimental Brain Research*, *133*(3), 407–413. https://doi.org/10.1007/s002210000410

Morgan, C. L. (1978). Constancy of egocentric visual direction. *Perception & Psychophysics*, *23*(1), 61–68. https://doi.org/10.3758/BF03214296

Nakayama, K. (1977). Geometric and physiological aspects of depth perception. *Three-Dimensional Imaging*, *0120*, 2–9. https://doi.org/10.1117/12.955728

Ogle, K. N. (1938). Induced size effect: I. A new phenomenon in binocular space perception associated with the relative sizes of the images of the two eyes. *Archives of Ophthalmology*, *20*(4), 604–623. https://doi.org/10.1001/archopht.1938.00850220076005

Ogle, K. N. (1950). *Researches in binocular vision*. Saunders.

Ogle, K. N. (1962a). The problem of the horopter. In H. Davson (Ed.), *The Eye: Volume 4 Visual Optics and the Optical Space Sense* (pp. 325–348). Academic Press.

Ogle, K. N. (1962b). A consideration of the horopter: the Proctor award lecture. *Investigative Ophthalmology & Visual Science*, *1*(4), 446–461.

O'leary, A., & Wallach, H. (1980). Familiar size and linear perspective as distance cues in stereoscopic depth constancy. *Perception & Psychophysics*, *27*(2), 131–135. https://doi.org/10.3758/BF03204300

Ono, H., & Comerford, J. (1977). Stereoscopic depth constancy. In W. Epstein (Ed.), *Stability and Constancy in Visual Perception: Mechanisms and Processes* (pp. 91–128). Wiley.

Palmisano, S., Gillam, B., Govan, D. G., Allison, R. S., & Harris, J. M. (2010). Stereoscopic perception of real depths at large distances. *Journal of Vision*, *10*(6), 19. https://doi.org/10.1167/10.6.19

Ritter, M. (1977). Effect of disparity and viewing distance on perceived depth. *Perception & Psychophysics*, *22*(4), 400–407. https://doi.org/10.3758/BF03199707

Rogers, B. J., & Bradshaw, M. F. (1993). Vertical disparities, differential perspective and binocular stereopsis. *Nature*, *361*(6409), 253–255. https://doi.org/10.1038/361253a0

Rogers, B. J., & Bradshaw, M. F. (1995). Disparity scaling and the perception of frontoparallel surfaces. *Perception*, *24*(2), 155–179. https://doi.org/10.1068/p240155

Rogers, B. J., & Cagenello, R. (1989). Disparity curvature and the perception of three-dimensional surfaces. *Nature*, *339*(6220), 135–137. https://doi.org/10.1038/339135a0

Schreiber, K. M., Hillis, J. M., Filippini, H. R., Schor, C. M., & Banks, M. S. (2008). The surface of the empirical horopter. *Journal of Vision*, *8*(3), 7. https://doi.org/10.1167/8.3.7

Sedgwick, H. A., & Gillam, B. (2017). A non-modular approach to visual space perception. *Ecological Psychology*, *29*(2), 72–94. https://doi.org/10.1080/10407413.2017.1297189

Shipley, T., Ogle, K. N., & Rawlings, S. C. (1970). The nonius horopter—I. History and theory. *Vision Research*, *10*(11), 1225–1262. https://doi.org/10.1016/0042-6989(70)90039-8

Stevenson, S. B., & Schor, C. M. (1997). Human stereo matching is not restricted to epipolar lines. *Vision Research*, *37*(19), 2717–2723. https://doi.org/10.1016/S0042-6989(97)00097-7

Swenson, H. A. (1932). The relative influence of accommodation and convergence in the judgment of distance. *The Journal of General Psychology*, *7*(2), 360–380. https://doi.org/10.1080/00221309.1932.9918473

von Helmholtz, H. (1962). *Helmholtz's treatise on physiological optics, Vol. 3 , Trans. from the 3rd German ed* (J. P. C. Southall, Trans.). Dover Publications.

Vreven, D. (2006). 3D shape discrimination using relative disparity derivatives. *Vision Research*, *46*(25), 4181–4192. https://doi.org/10.1016/j.visres.2006.08.014

Wallach, H., & Zuckerman, C. (1963). The constancy of stereoscopic depth. *The American Journal of Psychology*, *76*(3), 404–412. https://doi.org/10.2307/1419781

Wheatstone, C. (1838). Contributions to the physiology of vision. part the first. On some remarkable, and hitherto unobserved, phenomena of binocular vision. *Philosophical Transactions of the Royal Society of London*, *128*, 371–394. https://doi.org/10.1098/rstl.1838.0019